# Smile Dynamics and Rough Volatility

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joint work with Florian Bourgey (Bloomberg NY)
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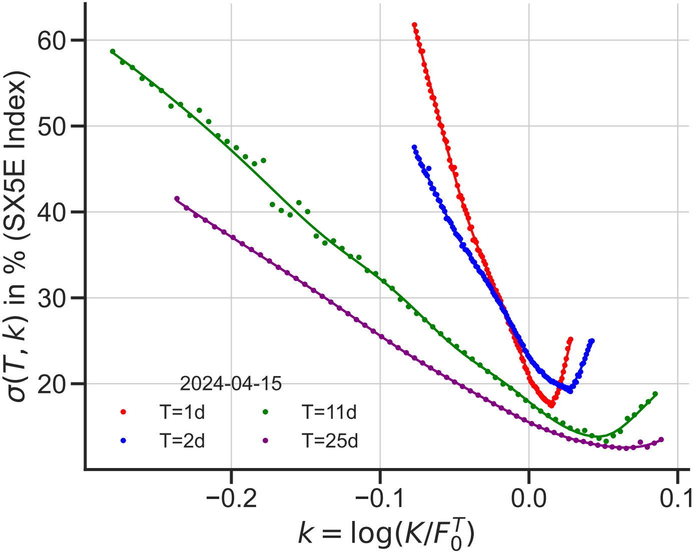
# Dynamics of the implied volatility surface



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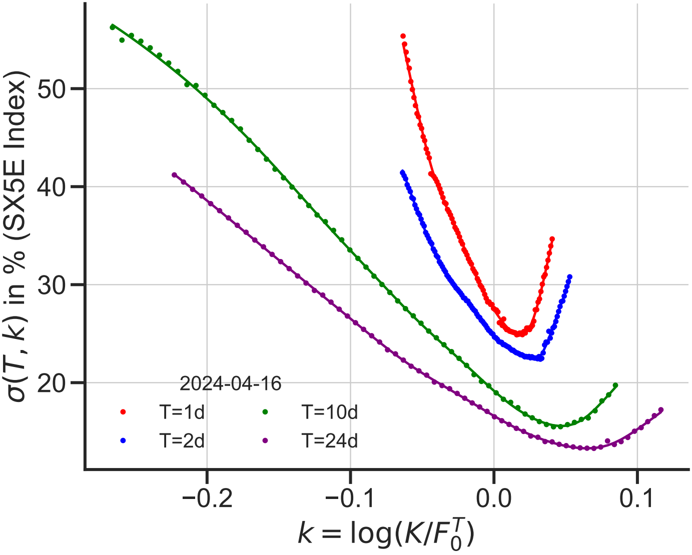

# Modeling with stochastic volatility

**Stochastic volatility models:**

- Large volume of literature assessing their **static** properties
  - Calibration accuracy to the implied vol surface
  - Specific features of the implied vol surface: e.g. term structure of the at-the-money skew

# Modeling with stochastic volatility

**Stochastic volatility models:**

- Large volume of literature assessing their **static** properties
  - Calibration accuracy to the implied vol surface
  - Specific features of the implied vol surface: e.g. term structure of the at-the-money skew
- Information is more scarce about their **dynamic** properties
  - How do the implied vols generated by the model move jointly with the spot price? (the SSR indicator)
  - How do implied volatilities for different strikes move together?

Famous series of papers by L. Bergomi (Soc Gen at the time): Smile Dynamics I, II, III and IV, 2004 to 2009.

# Dynamic properties of stochastic volatility models

Important implications for:

- PnL control when hedging exotics (e.g. Autocalls) with Vanilla options
- Model recalibration
- Volatility market making

► Different applications might require different models or different tools, and focus on different objects

► Focus of this talk: smile dynamics with a view on exotic option hedging

► We will be interested in implied volatilities, not realized volatilities.

# PnL representation: classical delta hedging

- **Delta hedge of an option (short position) on an asset $S$ with Black-Scholes formulae**

- Black-Scholes price $P_\sigma(t, S_t)$ and Black-Scholes delta $\partial_S P_\sigma(t, S_t)$, constant volatility parameter $\sigma$

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- **Classical theta-gamma PnL approximation** over $[t, t+\Delta t]$ (say $\Delta t = 1$ day):

$$\begin{aligned}\mathrm{PnL}_{t,t+\Delta t} &= -\big(P_\sigma(t+\Delta t, S_{t+\Delta t}) - P_\sigma(t, S_t)\big) \\ &\approx \frac{1}{2} S_t^2 \, \frac{\partial^2 P_\sigma(t, S_t)}{\partial S^2} \left[\sigma^2 \, \Delta t - \left(\frac{\delta S_t}{S_t}\right)^2\right]\end{aligned}$$

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- Model parameter $\sigma^2$ against market value of realized squared return $\left(\frac{\delta S_t}{S_t}\right)^2$
- Setting $\sigma^2 = \mathbb{E}_{\mathbb{P}^{\mathrm{hist}}}\left[\left(\frac{\delta S_t}{S_t}\right)^2 \middle| \mathcal{F}_t\right]$ yields $\mathbb{E}_{\mathbb{P}^{\mathrm{hist}}}[\mathrm{PnL}_{t,t+\Delta t} | \mathcal{F}_t] = 0$.

# PnL representation: delta and vega hedging

- **Hedging an exotic option with the underlying and Vanilla options** (with their implied volatilities $\hat{\sigma}_{KT}$)

- Using a Local Stochastic Volatility (LSV) model

$$\frac{\mathrm{d}S_t}{S_t} = l(t, S_t)\sqrt{V_t}\,\mathrm{d}W_t^S, \quad V_t \text{ an autonomous process}$$

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- **PnL decomposition** ([Bergomi, Risk 2017] also in [Bergomi 2016, Chap 12])

$$\begin{aligned}\mathrm{PnL}_{t,t+\Delta t} \approx\ & \frac{1}{2}\frac{\partial^2 P}{\partial S^2}\left[l(t,S_t)^2 V_t\,\Delta t - \left(\frac{\delta S_t}{S_t}\right)^2\right] \\ & + S_t \sum_{T,K} \frac{\partial P}{\partial S\,\partial\hat{\sigma}_{KT}}\left[\frac{d\langle \log S, \sigma(T,K)\rangle_t}{dt}\,\Delta t - \frac{\delta S_t}{S_t}\,\delta\hat{\sigma}_{KT}\right] \\ & + \frac{1}{2}\sum_{T,K,T',K'} \frac{\partial^2 P}{\partial\hat{\sigma}_{KT}\,\partial\hat{\sigma}_{K'T'}}\left[\frac{d\langle \sigma(T,K), \sigma(T',K')\rangle_t}{dt}\,\Delta t - \delta\hat{\sigma}_{KT}\,\delta\hat{\sigma}_{K'T'}\right],\end{aligned}$$

where $\sigma(T,K)$ is the model implied vol and $\hat{\sigma}_{KT}$ the market implied vol.

- Same interplay: model implied quantities against market mouvements.

# Static & dynamic properties of stoch vol models (again)

- Spot-vol dynamics: encoded by the Itô covariations $\langle \log(S), \sigma(T,K)\rangle_t$
- Vol-vol dynamics: encoded by the Itô covariations $\langle \sigma(T,K), \sigma(T',K')\rangle_t$

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## What we can ask for

To be able to set the model parameters so that

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- Multi-factor stochastic vol models (e.g. 2-factor Bergomi model) and their LSV corrections have been designed to do so.
- Rough volatility? Any advantage at this level?

  ► We focus on an indicator of joint spot-vol dynamics: the Skew-Stickiness Ratio.

From now on, let us denote

- Implied volatility of call/put options for log-forward moneyness $k = \log \frac{K}{F_t^{t+\tau}}$ and **time to maturity** $\tau$

$$\sigma_t(\tau, k)$$

- **At-the-money forward (ATMF) implied volatility** for time to maturity $\tau$

$$\sigma_t(\tau) = \sigma\big(\tau, k\big)\big|_{k=0}$$

- **ATMF implied volatility skew** for time to maturity $\tau$

$$\mathcal{S}_t(\tau) = \partial_k \sigma_t\big(\tau, k\big)\big|_{k=0}$$

# Joint increments of log-price and implied vol, $\tau = 1m$

$$\delta \log S_t = \log S_{t+\Delta} - \log S_t \qquad \delta\sigma_t(\tau) = \sigma_{t+\Delta}(\tau) - \sigma_t(\tau) \qquad \Delta = 1 \text{ day}$$

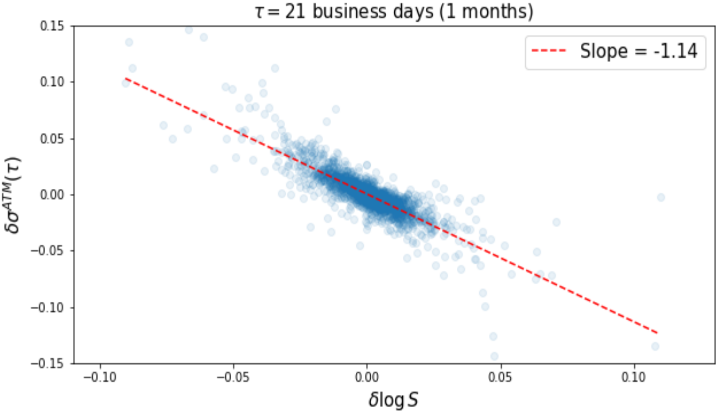


Figure: Joint daily increments $(\delta \log S, \delta\sigma(\tau))$ for the SPX over 2008-2016, $\tau = 1$ month.

$$\text{slope} = \beta(\tau) = \frac{\sum_{i=1}^{n} \delta \log S_{t_i}\, \delta\sigma_{t_i}(\tau)}{\sum_{i=1}^{n} (\delta \log S_{t_i})^2}$$

# Joint increments of log-price and implied vol, $\tau = 3m$

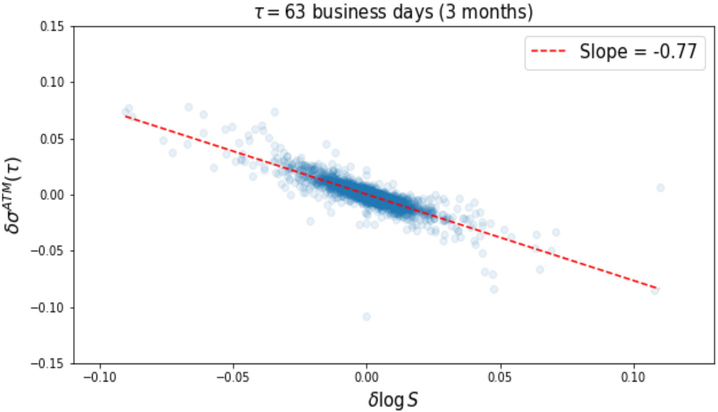


Figure: Joint increments $(\delta \log S, \delta\sigma(\tau))$ for the SPX over 2008-2016, $\tau = 3$ months.

# Joint increments of log-price and implied vol, $\tau = 6m$

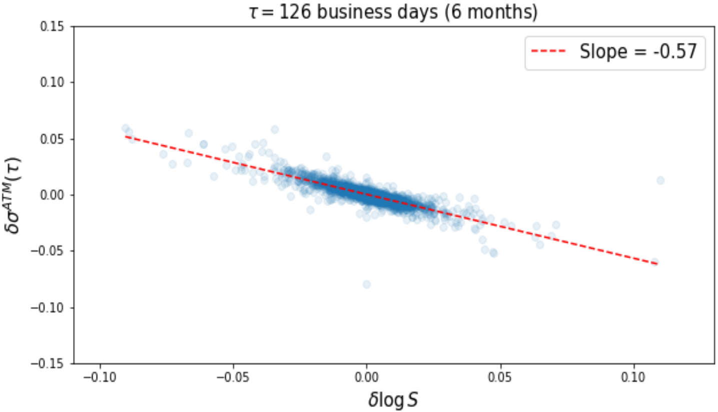


Figure: Joint increments $(\delta \log S, \delta\sigma(\tau))$ for the SPX over 2008-2016, $\tau = 6$ months.

# Joint increments of log-price and implied vol, $\tau = 1y$

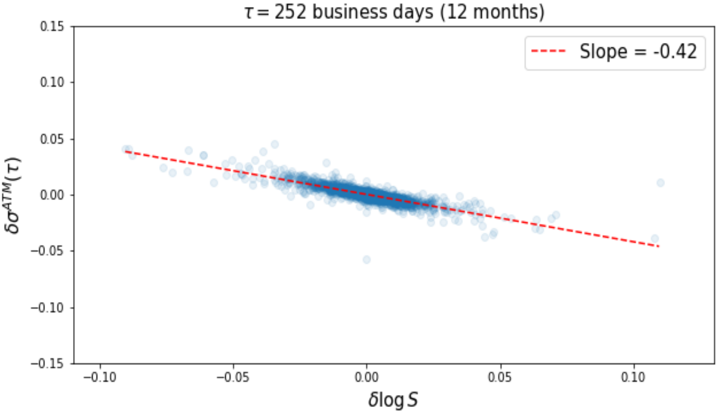
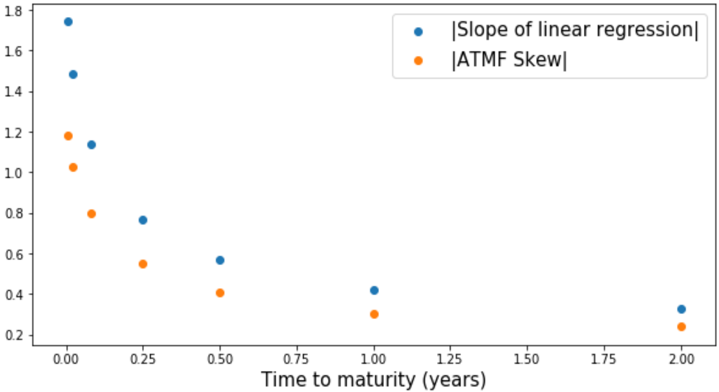


Figure: Joint increments $(\delta \log S, \delta\sigma(\tau))$ for the SPX over 2008-2016, $\tau = 1$ year.

# Slope vs. ATMF Skew



- Blue: slopes $\beta(\tau)$ of the previous linear regressions
- Orange: ATMF skew $\mathcal{S}(\tau)$ averaged over the same period

# The Skew-Stickiness ratio $\mathcal{R}(\tau)$

- **The Skew-Stickiness Ratio (SSR)** [Bergomi, *Smile Dynamics IV*, 2009] is precisely the ratio of the two quantities

$$\mathcal{R}(\tau) = \frac{\beta(\tau)}{\mathcal{S}(\tau)}$$

- A dimensionless number. Tells how the implied volatility reacts (locally around the ATM point) to a change of the spot as opposed to a change of the strike.

- Requires the skew $\mathcal{S}(\tau)$ to be non-zero (mostly well-suited for Equity markets, not really for FX or Commodities).

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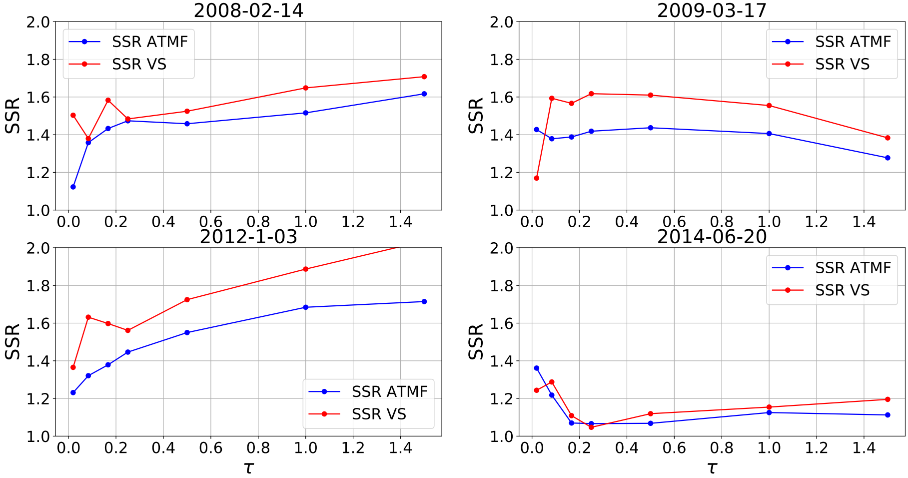
- Two classical dynamic regimes:
  - sticky delta $\mathcal{R}(\tau) = 0$
  - sticky strike $\mathcal{R}(\tau) = 1$

- Market SSR: estimated from market movements (daily or intra-day). Typically, noisy estimation.

# Market SSR: sample daily term structures for the SPX index



Sample term structures of Vanilla ATMF SSR and Var Swap SSR for the SPX index. From a time series of daily last traded vols. Rolling window $\ell = 63$ days.

# Model SSR

**SSR for the ATMF implied volatility of Vanillas**:

$$\mathcal{R}_t(\tau) := \frac{1}{\mathcal{S}_t(\tau)} \frac{\frac{\mathrm{d}}{\mathrm{dt}}\langle \log S, \sigma(\tau)\rangle_t}{\frac{\mathrm{d}}{\mathrm{dt}}\langle \log S\rangle_t}.$$

**SSR for the Variance Swap implied volatility**[1]:

$$\mathcal{R}_{\mathrm{VS},t}(\tau) := \frac{1}{\mathcal{S}_t(\tau)} \frac{\frac{\mathrm{d}}{\mathrm{dt}}\langle \log S, \sigma_{\mathrm{VS}}(\tau)\rangle_t}{\frac{\mathrm{d}}{\mathrm{dt}}\langle \log S\rangle_t}.$$

- [Bergomi 2009], [Bergomi 2016]: local volatility and Markovian stochastic volatility models generate

  $$\mathcal{R}(\tau) \approx \mathcal{R}_{\mathrm{VS},t}(\tau) \approx 2 \qquad \text{for small } \tau,$$

  together with model-specific term structures $(\mathcal{R}(\tau))_{\tau>0}$ and $(\mathcal{R}_{\mathrm{VS},t}(\tau))_{\tau>0}$.

---

[1] Recall that, in an Itô model, var swap implied vol = log-contract implied vol = implied vol of a basket of Vanillas.

# General forward variance model

A joint model for spot $S$ and the instantaneous forward variance curve $(\xi_t^u)_{u \geq t}$:

$$\frac{\mathrm{d}S_t}{S_t} = (r - q)\mathrm{d}t + \sqrt{\xi_t^t}\,\mathrm{d}W_t^S$$

$$\mathrm{d}\xi_t^u = \lambda\big(t, u, \xi_t^{v(t,u)}\big) \cdot \mathrm{d}W_t$$

- $W_t^S$: a one-dim Brownian motion.
- $W_t \in \mathbb{R}^n$: $n$-dim Brownian motion correlated with $W^S$, $\frac{\mathrm{d}}{\mathrm{d}t}\langle W^S, W\rangle_t = \rho_{S\xi}(t)$.
- $\Big(\lambda\big(t, u, \xi_t^{v(t,u)}\big)\Big)_{t \leq u}$: a $n$-dimensional stochastic process (volatility of variance)

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**The instantaneous forward variance $\xi_t^u$:**

- Related to the Variance Swap implied variance $\sigma_{\mathrm{VS}}^2$ via

$$\frac{1}{\tau}\int_t^{t+\tau} \xi_t^u\,\mathrm{du} = \sigma_{\mathrm{VS,t}}^2(\tau)$$

  In the model, $\sigma_{\mathrm{VS}}$ is also the implied vol of the log-contract (the VIX)

- Related to the asset instantaneous variance $V$ via $\xi_t^u = \mathbb{E}[V_u|\mathcal{F}_t]$.

# Example: the 2-factor Bergomi model—a Markovian model

- **The two-factor Bergomi model (2fB)** [Bergomi 2005], see also [Dupire 93]

$$\frac{\mathrm{d}S_t}{S_t} = (r-q)\mathrm{d}t + \sqrt{\xi_t^t}\,\mathrm{d}W_t^S$$

$$\mathrm{d}\xi_t^u = \omega\,\alpha_\theta\,\xi_t^u\Big((1-\theta)e^{-k_1(u-t)}\mathrm{d}W_t^1 + \theta e^{-k_2(u-t)}\mathrm{d}W_t^2\Big)$$

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- Enjoys a 2-dimensional Markovian representation of the variance curve:

$$\xi_t^u = F(t,u,X_t^1,X_t^2), \qquad 0 \le t \le u,$$

where $X^1, X^2$ are two Ornstein–Uhlenbeck processes

$$\begin{cases} \mathrm{d}X_t^1 = -k_1 X_t^1 \mathrm{d}t + \mathrm{d}W_t^1, & X_0^1 = 0, \\ \mathrm{d}X_t^2 = -k_2 X_t^2 \mathrm{d}t + \mathrm{d}W_t^2 & X_0^2 = 0\,. \end{cases}$$

# Other examples: Rough volatility

- **The rough Bergomi model** [Bayer, Friz, and Gatheral 2016]:

$$\begin{aligned}\frac{\mathrm{d}S_t}{S_t} &= (r-q)\mathrm{d}t + \sqrt{\xi_t^t}\,\mathrm{d}W_t^S,\\ \mathrm{d}\xi_t^u &= \eta\sqrt{2H}\,\xi_t^u\,(u-t)^{H-\frac{1}{2}}\mathrm{d}W_t, \qquad \mathrm{d}\langle W^S, W\rangle_t = \rho\,\mathrm{dt}.\end{aligned}$$

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- **The rough Heston model** [El Euch and Rosenbaum 2019]:

$$\frac{\mathrm{d}S_t}{S_t} = (r-q)\mathrm{d}t + \sqrt{\xi_t^t}\,\mathrm{d}W_t^S,$$

$$\mathrm{d}\xi_t^u = \eta\sqrt{\xi_t^t}\,(u-t)^{H-\frac{1}{2}}\;E_{H+\frac{1}{2},H+\frac{1}{2}}\big(-\kappa(u-t)^{H+\frac{1}{2}}\big)\,\mathrm{d}W_t,$$

  where $E_{\alpha,\beta}(z) = \sum_{n=0}^{\infty}\frac{z^n}{\Gamma(\alpha n+\beta)}$ is the Mittag–Leffler function

  - ▶ When $H = 1/2$, boils down to the classical Heston model.

# SSR under rough volatility: short-maturity limit

**Theorem** (Bourgey, Delemotte, DM 2024)

*In both the rough Bergomi model and the rough Heston model, we have*

$$\mathcal{R}_{\mathrm{VS}}(\tau) \to \frac{3}{2} + H \qquad \text{as } \tau \to 0.$$

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$$\mathcal{R}_{\mathrm{VS}}(\tau) \to \frac{3}{2} + H \qquad \textit{as } \tau \to 0.$$

- Consistent with [Friz and Gatheral, 2025], based on formal asymptotic expansions (forest expansions).
- Extended by [Fukasawa 2026] to $\mathcal{R}(\tau) \to \frac{3}{2} + H$ (here ATMF Vanilla SSR $\mathcal{R}$ instead of Var Swap SSR $\mathcal{R}_{\mathrm{VS}}$—more difficult) in a class of rough models.
- For a non-exploding kernel where $H = \frac{1}{2}$, recover

$$\mathcal{R}_{\mathrm{VS}}(\tau) \to 2 \qquad \text{as } \tau \to 0$$

# At-the-money SSR: numerical computation

- If finite-dimensional Markovian representation for forward variances:

  $$\xi_t^u = F\left(t, u, X_t\right) \qquad X_t = (X_t^i)_{1 \leq i \leq n} \text{ Markov process in } \mathbb{R}^n$$

  Then ATMF implied vols are also functions of $X$: $\sigma_t^{\mathrm{ATMF}}(\tau) = \sigma^\tau\left(t, X_t\right)$

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- **Finite difference estimator of the model SSR under finite-dim Markov representation**:

$$\begin{aligned}\mathcal{R}_t(\tau) &= \frac{1}{\mathcal{S}_t(\tau)\,\xi_t^t} \nabla_X \sigma^\tau(t, X_t) \cdot \mathcal{C}^{\log(S),X}(t) \\ &= \frac{1}{\mathcal{S}_t(\tau)} \lim_{h \to 0} \frac{1}{h} \left( \sigma^\tau\left(t, X_t + \frac{h}{\xi_t^t} \mathcal{C}^{\log(S)X}(t)\right) - \sigma^\tau(t, X_t) \right),\end{aligned}$$

where

$$\mathcal{C}^{\log(S)X}(t) := \left( \frac{\mathrm{d}}{\mathrm{d}t} \langle \log S, X^i \rangle_t \right)_{1 \le i \le n} \in \mathbb{R}^n.$$

- We only have to evaluate two ATMF implied volatilities, for different values of the Markov factors.

# At-the-money SSR: computation, general model

- General case (rough volatility models): ATMF implied vols depend on the whole forward variance curve $\xi_t$:

$$\sigma_t^{\mathrm{ATMF}}(\tau) = \sigma^\tau\Big(t, (\xi_t^v)_{v\in(t,t+\tau)}\Big), \qquad \forall\, t \geq 0,\ \forall\, \tau > 0.$$

- From the infinite dimensional Itô formula ([Da Prato and Zabczyk 2014], [Viens and Zhang 2019])

$$\mathcal{R}_t(\tau) = \frac{1}{\mathcal{S}_t(\tau)} \frac{1}{\sqrt{\xi_t^t}}\, D_{\xi_t}\sigma^\tau\big(t, \xi_t\big)\Big[\lambda\Big(t, \cdot, \xi_t^{v(t,\cdot)}\Big) \cdot \rho_{S\xi}(t)\Big]$$

  - ► $D_{\xi_t}\sigma^\tau\Big(t, \xi_t\Big)[y(\cdot)]$: Fréchet derivative of the implied volatility $\sigma^\tau(t, \xi_t)$ with respect to the current forward variance curve $\xi_t$, acting on the function $y(\cdot)$.

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- **Finite difference estimator of the model SSR**, general case:

$$\mathcal{R}_t^N(\tau) = \frac{1}{\mathcal{S}_t(\tau)} \lim_{h\to 0} \frac{1}{h}\Bigg(\sigma^{\tau,N}\Bigg(t, \Bigg(\xi_t^{u_i} + h\frac{\lambda\big(t, u_i, \xi_t^{v(t,u_i)}\big) \cdot \rho_{S\xi}(t)}{\sqrt{\xi_t^t}}\Bigg)^{t<u_i<t+\tau}\Bigg) - \sigma^{\tau,N}\Big(t, \big(\xi_t^{u_i}\big)^{t<u_i<t+\tau}\Big)\Bigg).$$

with a time discretization over $N$ steps.

# Comparison of calibrated models (May 6, 2024)

- We calibrate several models to the same SPX market term structure (ATMF vol and ATMF skew) on a given date, and compare the SSR they generate.
- **A date with an increasing volatility term structure**: May 6, 2024, market with low vol regime.

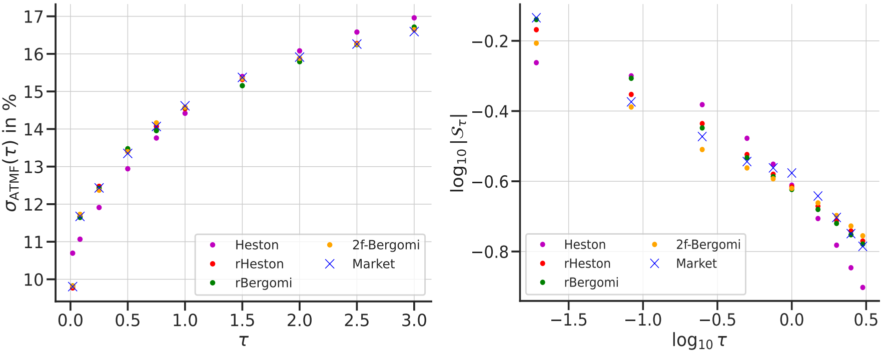

Figure: ATMF implied vols (left) and ATMF skews (right). Date: May 6, 2024.

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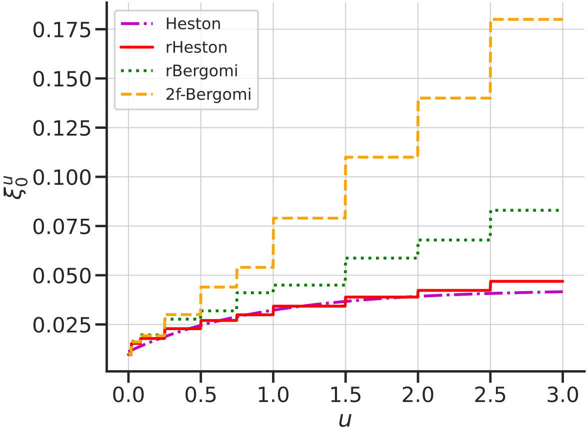

Figure: Calibrated initial forward variance curves $\xi_0$. Date: May 6, 2024.

# Comparison of calibrated models: SSR (May 6, 2024)

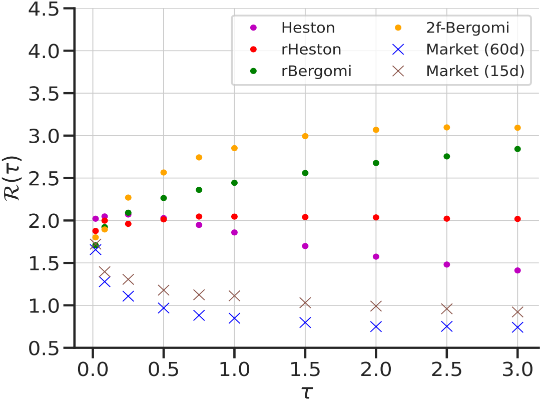

Figure: Dots: SSR of the calibrated models. Crosses: market SSR estimated over the past 60 or 15 days.

| Model | Calibrated parameters (6 May 2024) |
|---|---|
| Heston | $V_0 = 0.011$, $\kappa = 1.1$, $\overline{V} = 0.043$, $\eta = 0.24$, $\rho = -1$ |
| rHeston | $H = 0.38$, $\kappa = 3.44 \times 10^{-4}$, $\eta = 0.16$, $\rho = -0.91$ |
| 2f Bergomi | $\omega = 5.0$, $k_1 = 100$, $k_2 = 0.30$, $\theta = 0.36$, $\rho_{12} = 0.222$, $\rho_{S1} = -0.613$, $\rho_{S2} = -0.498$ |
| rBergomi | $H = 0.23$, $\eta = 1.50$, $\rho = -0.62$ |

# Comparison of calibrated models (April 7, 2025)

- **A date with a decreasing volatility term structure**: April 7, 2025, high vol regime (after the announcement of new US tariffs).

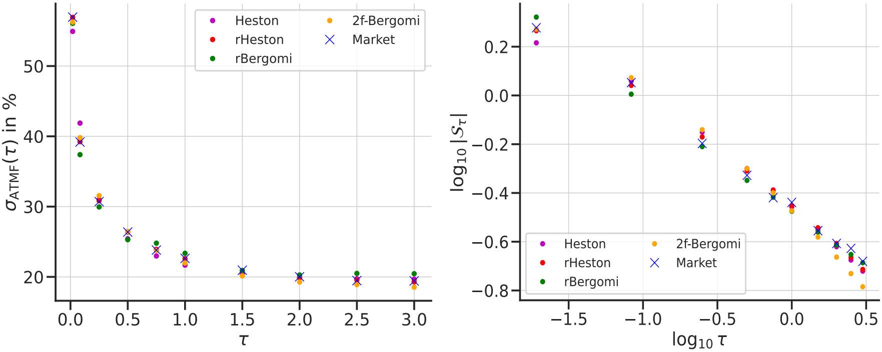

Figure: ATMF implied vols (left) and ATMF skews (right). Date: April 7, 2025.

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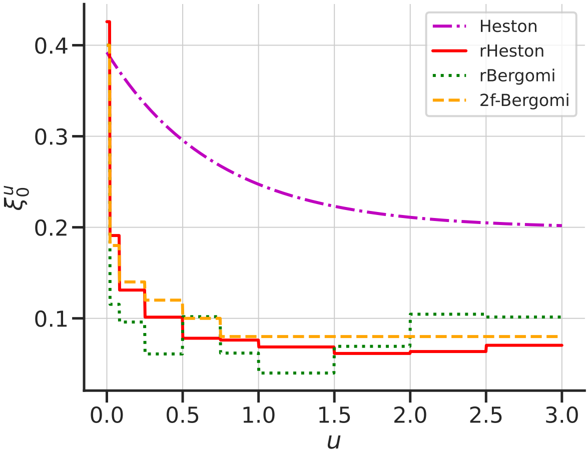

Figure: Calibrated initial forward variance curves $\xi_0$. Date: April 7, 2025.

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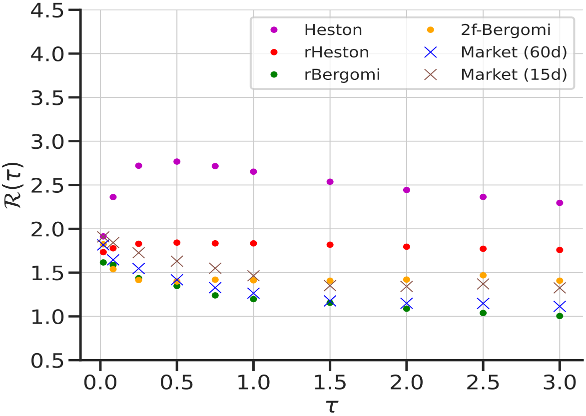

Figure: Dots: SSR of the calibrated models. Crosses: market SSR estimated over the past 60 or 15 days.

| Models | Calibrated parameters (April 7, 2025) |
|---|---|
| Heston | $V_0 = 0.39$, $\kappa = 1.38$, $\overline{V} = 0.20$, $\eta = 7.71$, $\rho = -0.48$ |
| rHeston | $H = 0.024$, $\kappa = 0.43$, $\eta = 0.99$, $\rho = -0.62$ |
| 2f Bergomi | $\omega = 11.5$, $k_1 = 24.5$, $k_2 = 2.1$, $\theta = 0.31$, $\rho_{12} = -0.70$, $\rho_{S1} = -0.47$, $\rho_{S2} = -0.30$ |
| rBergomi | $H = 0.097$, $\eta = 2.08$, $\rho = -0.99$ |

# Observations

- After calibration to SPX data, **all the models considered**, both Markovian and rough, **yield SSRs that are broadly similar**.
- This suggests an inherent rigidity within the Heston and Bergomi model classes, and indicates that rough volatility has a limited effect on joint spot-smile dynamics.
- The calibrated models can reproduce the empirical spot-implied volatility covariances around the calibration date (as on April 7, 2025), or fail to do so (as on May 6, 2024).
- In particular, on a date representative of an unstressed, low-volatility market (April 7, 2025), all models generate SSR levels and term structures that deviate from empirical market behavior.

# Conclusions

- **While rough volatility models drastically modify the dynamics of the instantaneous volatility (from Brownian motion to fractional Brownian motion), they do not seem to considerably alter the dynamics of implied volatilities**.

- Our analysis is (so far) limited to a small number of dates; a more extensive study on several dates (years) would be desirable.

- Of course, other models not within the families we have considered can still have better performances:
  - e.g. the Quintic OU model of [Abi Jaber and Li, Risk 2026]
  - Multi-factor path-dependent volatility?
  - ...more to come!

## References

Talk based on:

- Bourgey, F., Delemotte, J., and De Marco, S., (2024). *Smile dynamics and rough volatility*. SSRN 4911186.

- Bourgey, F., Delemotte, J., and De Marco, S. (2025). *Refined Expansions of the Skew-Stickiness Ratio in Stochastic Volatility Models*. SSRN 5387754.

- GitHub repository: `https://github.com/fbourgey/ssr`

Thank you