Inelastic Markets & “Ponzi Funds”

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Why do prices move? EMH* vs IMH**

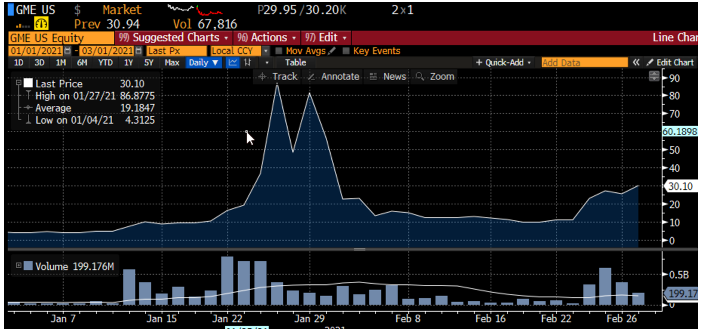
* Efficient Market Hypothesis
** Inelastic Market Hypothesis

Why do prices move? (An age-old question)

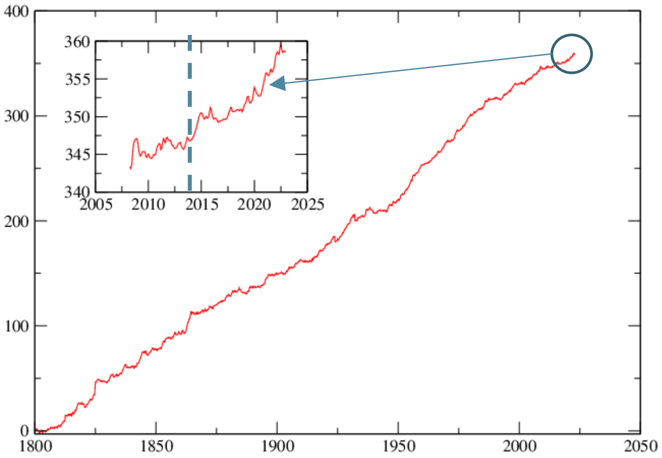
- EMH* says: because of unanticipated exogenous news
    - → Most financial economists speak about the “fundamental” price, that is “*discovered*” by markets
- Is this picture of markets even a **valid rough approximation**?

A slew of market "anomalies" at odds with EMH

- The excess volatility puzzle
  - → Most "jumps" are not due to news (Cutler-Poterba-Summers, CFM)
- The trend following puzzle + (many many) other profitable "anomalies":
  - → Prices do not reflect all available information
- Bitcoin and other fancy tulips – but "bubbles do not exist" (Fama)
- The "Reddit stocks" episodes, etc. etc.



Source: Bloomberg and CFM



6 month trends: Some (obvious) public information is not included in the current price!

- Note 1: no long bias here, but no costs either
- Note 2: 2024, a record year, 2026 roaring up

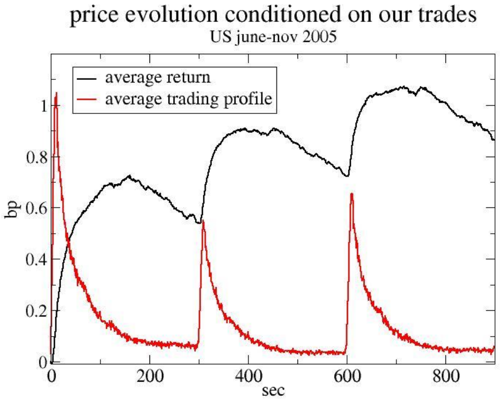
See: *Two centuries of Trend Following (2014)*

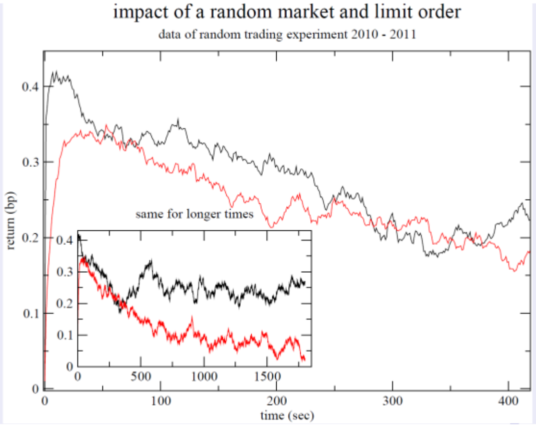
Why do prices move? (An age-old question)

- The Inelastic Market Hypothesis/Order-driven view of markets:
  - → Flows **impact** prices…and price changes alter flows
  - → Price "*formation*" rather than price "*discovery*"
- Such a change of paradigm has far-reaching consequences on economic theory and on the investment industry (cf. Keynes, Shiller, Soros, etc.)

Flow do impact prices: a (not so trivial) truism

- Even though for each buyer there is a seller, trading impacts prices (“active” vs. “passive” flows breaks the symmetry)
- The EMH view: it is simply because informed traders predict prices!
- The quant view: it is a “**mechanical**”, order flow effect even when trades are not informed (even random)
- (Impact is actually a major source of trading costs)





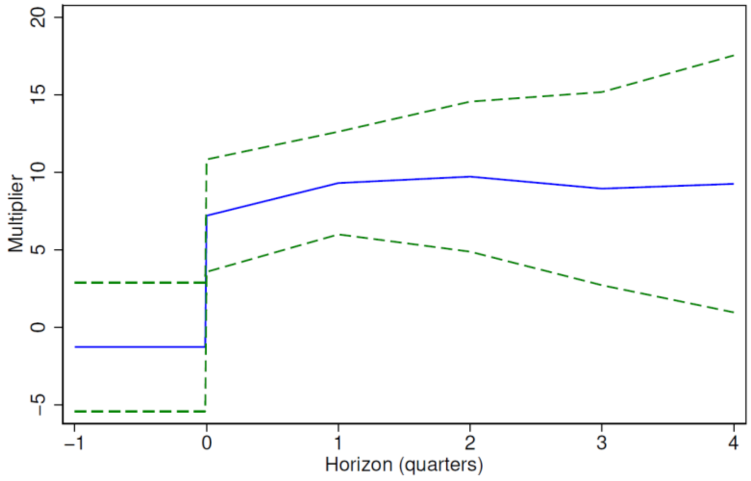
Source: CFM

Flows impact prices on the long run

Flow driven prices: a long-term phenomenon!

- OK, say EMH diehards, but impact decays “quickly” anyway and has no long-term consequences – really?
- The “Inelastic Market Hypothesis” (Gabaix-Koijen)
- Randomly buying (/selling) 1$ of the whole market increases (/decreases) the long-term market cap. by 5$!
- Mechanism: funds are inelastic due to e.g. mandate constraints -- say 80% stocks and 20% cash

Gabaix-Koijen Multiplier M=5



Source: CFM

Flow driven prices: a long-term phenomenon!

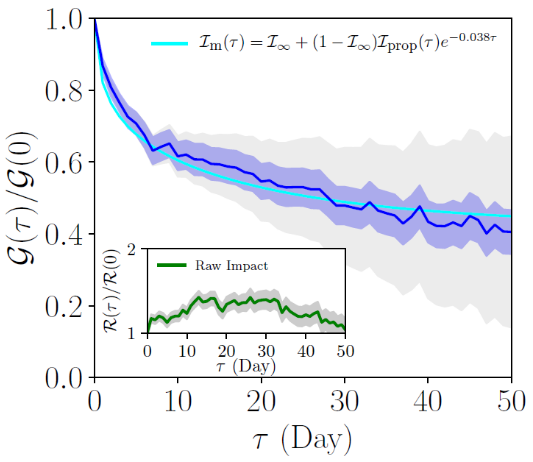
- The Gabaix-Koijen multiplier is more universal than imagined by GK – it is the long-term fate of the well-known square-root impact law (JPB, *Quantitative Finance* 2022)

Daily vol.

$$\boxed{M = \frac{1}{2}\sigma_1 \frac{\mathcal{M}}{V_1} \times \sqrt{\frac{1}{T_\mathrm{m}}}.}$$

Memory time

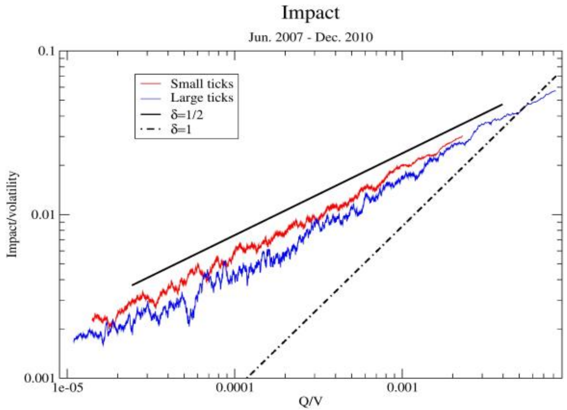
Market cap./ADV



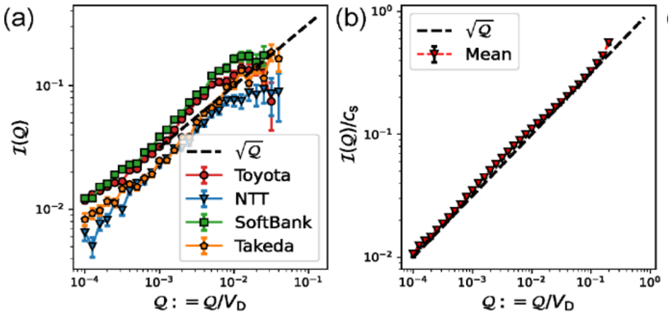
Stocks (Ancerno data)

Remark: the sqrt impact law

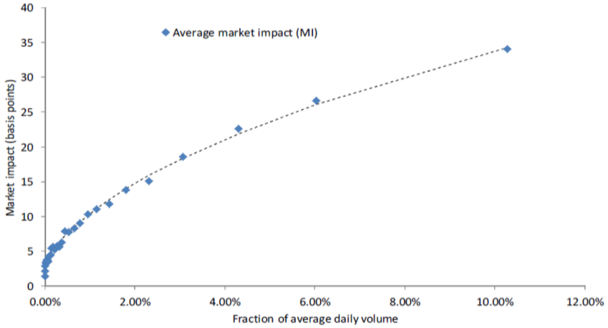
$$I(Q) = Y\sigma_T\sqrt{\frac{Q}{V_T}}$$



Futures (CFM data)



Stocks (TSE,
Sato Kanazawa)



Stocks (AQR data,
limit orders)

The GK multiplier as the long term fate of sqrt impact (LLOB theory, with M. Benzaquen)

The general solution of Eq. (8) is given by:

$$\phi(x,t) = (\mathcal{G}_\nu * \phi_0)(x,t) + \int \mathrm{d}y \int_0^\infty \mathrm{d}\tau\, \mathcal{G}_\nu(x-y, t-\tau) s(y,\tau)\,, \tag{9}$$

where $\phi_0(x) = \phi(x,0)$ denotes the initial condition, and

$$\mathcal{G}_\nu(x,t) = \max(t,0) \frac{\exp\left[-\frac{x^2}{4\sigma_1^2 t} - \nu t\right]}{\sqrt{4\pi\sigma_1^2 t}}\,. \tag{10}$$

Following Donier *et al.* [44], we introduce a buy (sell) meta-order as an extra point-like source of buy (sell) particles with intensity rate $m = Q/T$, where Q is the volume of the metaorder and T the execution time, such that the source term in Eq. (8) becomes: $s(x,t) = m\delta(x - x_t)\cdot \mathbb{1}_{[0,T]} + \lambda\, \mathrm{sign}(x_t - x)$.

Performing the integral over space in Eq. (9) and setting $\phi_0(x) = \phi^{\mathrm{st}}(x)$ yields:

$$\phi(x,t) = \phi^{\mathrm{st}}(x) e^{-\nu t} + m \int_0^{t\wedge T} \mathrm{d}\tau\, \mathcal{G}_\nu(x - x_\tau, t-\tau) - \lambda \int_0^t \mathrm{d}\tau\, \mathrm{erf}\left[\frac{x - x_\tau}{\sqrt{4D(t-\tau)}}\right] e^{-\nu(t-\tau)}\,. \tag{11}$$

The price x_t solves the integral equation:

$$\phi(x_t, t) = 0\,. \tag{12}$$

For $\lambda, \nu \to 0$ and for $t > T$, one immediately recovers Eq. (16) of [44]:

$$x_t = x_t^0 = \frac{m}{\mathcal{L}} \int_0^T \mathrm{d}\tau\, \mathcal{G}_0(x_t - x_\tau, t-\tau), \tag{13}$$

which boils down, at large t, to

$$x_t^0 \approx \frac{Q}{\mathcal{L}} \frac{1}{\sqrt{4\pi\sigma_1^2 t}} = \frac{\sigma_1}{\sqrt{4\pi t}} \frac{Q}{V_1}. \tag{14}$$

Setting $t = T_{\mathrm{m}}$ in this equation immediately leads to Eq. (2), up to a numerical prefactor.

In order to compute the long term impact exactly, the main idea of the calculation is to expand th price trajectory x_t in powers of $\sqrt{\nu}$, i.e.

$$x_t = x_t^0 + \sqrt{\nu} x_t^1 + O(\nu), \tag{15}$$

where x_t^0 and x_t^1 respectively denote the 0th order and 1st order contributions. In the limit of shor execution times ($T \ll T_{\mathrm{m}}$) and small meta-order volumes $Q \ll V_{\mathrm{m}}$, where $V_{\mathrm{m}} = V_1 T_{\mathrm{m}}$ is the total volum traded during the memory time T_{m}, one can look for a solution of the form $x_t^1 = F(\nu t)$. In the long tim limit $t \gg T$, using the zero-th order solution Eq. 14 and setting $u = \nu t$, Eq. (11) boils down to

$$0 = F(u) + \beta \int_0^u \mathrm{d}v \frac{\sqrt{v} - \sqrt{u}}{\sqrt{\pi u v (u-v)}} e^v + \int_0^u \mathrm{d}v \frac{F(u) - F(v)}{\sqrt{\pi(u-v)}} e^v\,, \tag{16}$$

where β depends on the fast/slow nature of the execution (see [46] for more details). The solution c this equation for $u \gg 1$ can be found to be

$$F(u) = F_\infty - \frac{\beta}{\sqrt{u}}\left[1 - e^{-u}\right]\,, \tag{17}$$

ETF as a testbed for the theory

ETF: A perfect testbed

- ETF: investment funds (both passive and active) that track **baskets of securities**
- ETFs have to file their holdings daily (GK study on mutual funds uses quarterly frequency)
- $12 Trillion under management, thousands of ETFs
- Demand for ETF **propagates to underlying security prices** via arbitrage mechanism

Our findings:

Inflows in ETF $\xrightarrow{\textbf{Impact}}$ ETF price increases $\xrightarrow{\textbf{Trend chasing}}$ Inflows in ETF

In collaboration with:
Philippe Van Der Beck (Harvard Business School)
Dario Villamaina (CFM)

Impact of rebalancing trades on ETF returns

- **Demand for ETF propagates to underlying security prices**

$$Flow_t^{ETF} \equiv \frac{\Delta \text{Share}_t^{ETF}}{\text{Share}_{t-1}^{ETF}}$$

Shares of ETF created at time t (in percentages)

$$\Delta \text{Share}_t^{ETF} \frac{\text{Ownership}_{t-1}^{ETF}(s)}{\text{Share}_{t-1}^{ETF}} \equiv Flow_t^{ETF} \text{Ownership}_{t-1}^{ETF}(s)$$

Demand induced on stock ***s*** by ETF share issuance

$$\text{Impact}_t^{ETF} \equiv Flow_t^{ETF} \left\langle \frac{\text{Ownership}_{t-1}^{ETF}(s)}{\text{Volume}_t(s)} \right\rangle$$

Averaged over all stocks held by the ETF

Illiquidity of underlying basket

A regression model

Demand for ETF propagates to underlying security prices and **in turn impacts ETF returns**

$$\text{Impact}_t^{ETF} \equiv Flow_t^{ETF} \times \mathcal{I}_t^{ETF} \rightarrow$$

Illiquidity of underlying basket

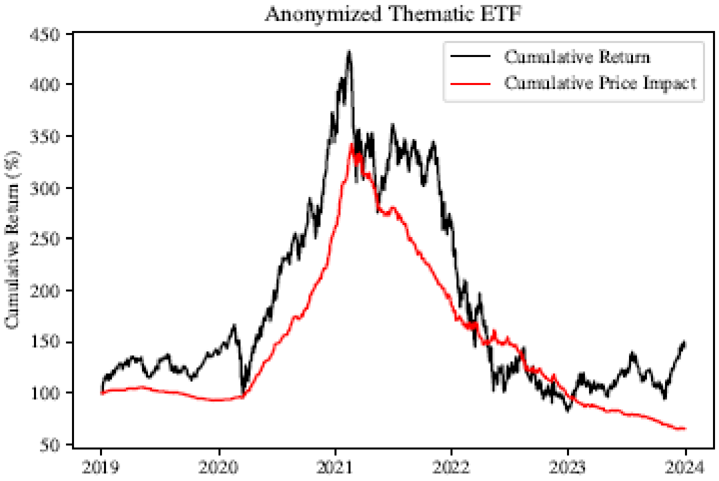
$$R_t^{ETF} = \underbrace{\beta \times \text{Impact}_t^{ETF}}_{\textbf{Price impact}} + \text{Controls} + \epsilon_t^{ETF} + \text{co-impact}$$

$$\underbrace{\Theta_F \times Flow_t^{ETF}}_{\textbf{“Information”}} + \underbrace{\Theta_I \times \mathcal{I}_t^{ETF}}_{\textbf{Illiquidity premium}}$$

Source: CFM

A regression model: results and illustration

	Excess ETF Return				
	(1)	(2)	(3)	(4)	(5)
$f \times \mathcal{I}$		0.085***	0.084***	0.086***	0.078***
		(0.019)	(0.019)	(0.019)	(0.017)
Co-Impact				0.052***	0.050***
				(0.009)	(0.009)
Flow f	0.005*	0.000	0.000	0.000	0.000
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Fund Illiquidity $\mathcal{I}$		-0.000*	-0.000**	-0.000***	-0.000**
		(0.000)	(0.000)	(0.000)	(0.000)



« Self-inflated returns »

Source: CFM

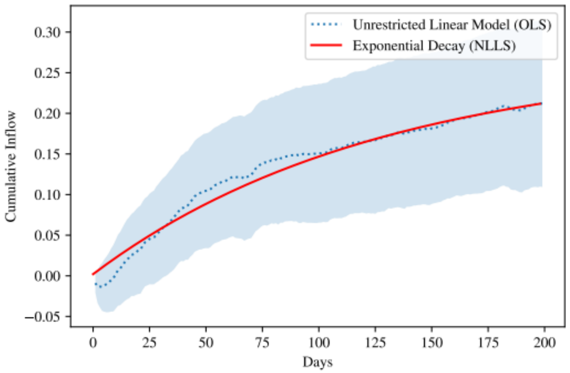
Performance chasing

Performance predicts future flows….

- Trend chasing behaviour + persistent flows

$$\text{Flow}_{t+1}^{ETF} = \sum_{l=0}^{200} \left[\beta_{t-l}^{R} R_{t-l}^{ETF} + \beta_{t-l}^{F} \text{Flow}_{t-l}^{ETF}\right] + \epsilon_{t+1}^{ETF}$$

Cumulative sum of return coefficients



Lagged days

Source: CFM

But investors do not distinguish skill from impact!

$$f^i_{t+1} = \alpha_t + \beta_1 R^{i,\perp}_{\lambda,t} + \beta_2 R^{i,\mathcal{I}}_{\lambda,t} + Controls + \epsilon^i_{t+1}$$

- Can investors distinguish skill from impact $\beta_1 \neq \beta_2$?

	(1)	(2)
R	0.2136***	
	(3.4301)	
$R^{\mathcal{I}}$		0.2375***
		(3.5943)
$R^{\perp}$		0.2229***
		(3.3425)
Controls	Yes	Yes
Effects	Time	Time
No. Obs.	1139565	1139565
R-squared	0.0150	0.0150

Investors chase both self-inflated and fundamental returns

1% self-inflated returns lead to 0.2% higher flows!

- **“Ponzi funds”**

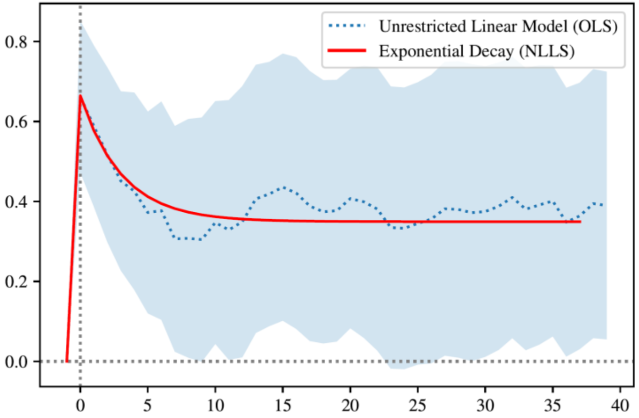
Source: CFM

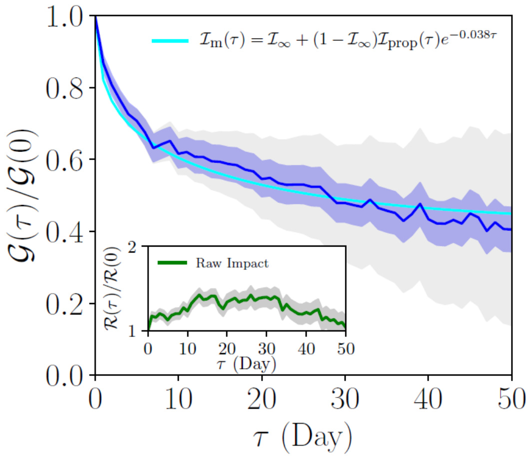
Impact: a universal effect

Impact estimates are in line with previous measures

- Previous regressions are even better with a square-root impact model with standard parameters

$$\frac{r_t(s)}{\sigma_t(s)} = \sum_{t'=0}^{T} \Theta_{t'} \cdot \sqrt{\mathrm{Impact}_{t-t'}(s)} + \mathrm{controls} + \epsilon_t(s)$$





Source: CFM

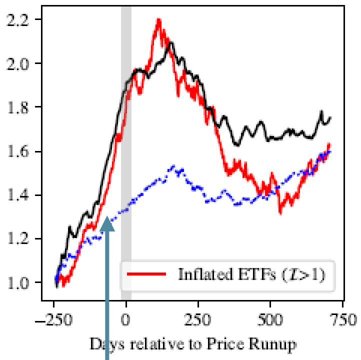
Take-home messages

IMH explains much more than EMH!

The theory explains (most?) markets anomalies

- On short to medium time scales, flows are an important determinant of price moves – independently of why people trade
- If money flows into some funds or into the market as a whole, prices will increase – for no other reason
  (orders of magnitude match for the 2009-2021 market run)
- Investors do not distinguish between skill and impact: direct evidence of self-sustained trends
- As appetite diminishes, impact relaxes and performance goes south: a scenario explaining the boom and bust of trends in alternative (illiquid) markets
- Prediction markets suffer from the same predicament
- Estimates of self-inflated CFM stat-arb returns: negligible

(b) Fund Illiquidity $\mathcal{I}_{i,t} > 1$



Market

Source: D. Villamaina, Ph. van der Beck, J.P.B. (2024)